Replacement Sensor Model Tagged Record Extensions Specification for NITF 2.1

APPENDIX B

draft

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RSM Partial Derivatives

Introduction

This appendix presents the analytic partial derivatives of the RSM adjustable ground-to-image function with respect to the ground location and also with respect to the RSM adjustable parameters. The RSM ground-to-image function may be either a (rational) polynomial or an interpolated ground point-image point correspondence grid. The polynomial is defined in the RSMPCA TRE, and the grid is defined in the RSMGGA TRE.

(Note that, in lieu of analytic partial derivative, numerical partial derivatives can be used. They are straightforward to define for the RSM application, but generally require more time to compute and are generally less accurate than analytic partial derivatives. Section 4.1.1 of the "Replacement Sensor Model", contained in Appendix C, describes numerical partial derivatives for the RSM application in more detail.)

Let us first define the following:

X is the three dimensional ground point represented in the RSM primary ground coordinate system (3×1);
R is the RSM (active) adjustable parameters ($m\times1$);
$g(X)$ is the ground-to-image function;
$h(X,R)$ is the adjustable ground-to-image function;
$I=\begin{bmatrix} r & c \end{bmatrix}^T$ is the image location (2×1);
$\Delta I=\begin{bmatrix} \Delta r & \Delta c \end{bmatrix}^T$ is the summed effect of all RSM "image-space" adjustable parameters;
X^* is X represented in the Local coordinate system;
ΔX^* is the summed effect of all RSM "ground-space" adjustable parameters;
$X_a^*=X^*+\Delta X^*$ is the adjusted ground point represented in the Local system;
X_a is the corresponding adjusted ground point represented in the RSM primary ground coordinate system;
$X_a=X+\Delta X$ defines the adjustment ΔX in the RSM primary ground coordinate system.

Note that:

ΔI is a function of X^* and R (active image-space adjustable parameters).
ΔX^* is a function of $X*$ and R (active ground-space adjustable parameters).

The RSM primary ground coordinate system is defined in the RSMIDA TRE. The RSM adjustable parameters, their summed effects, and the Local coordinate system are defined (redundantly) in the RSMDCA, RSMAPA, and RSMECA TRE's.

General form of partial derivatives associated with image-space adjustable parameters

$$I = h(X, R) = g(X) + \Delta I$$

$$\partial I / \partial R = \partial g / \partial R + \partial \Delta I / \partial R = 0 + \partial \Delta I / \partial R$$

$$\partial I / \partial R = \partial \Delta I / \partial R \qquad (2 \times m) \qquad \text{(B-1)}$$

Note that the partial derivative $\partial \Delta I / \partial R$ will be detailed later in equation (B-5), and is a function of X^* only.

$$\partial I / \partial X = \partial g / \partial X + [\partial \Delta I / \partial X^*][\partial X^* / dX] \qquad (2 \times 3) \qquad \text{(B-2)}$$

Note that $\partial g / \partial X$ is the partial derivative of the RSM ground-to-image function with respect to the ground point. The RSM ground-to-image function $g(X)$ is either a (rational) polynomial, or an interpolated ground point-image point correspondence grid, or their sum. Therefore, $\partial g / \partial X$ is either the partial derivative of a (rational) polynomial with respect to the ground point, or the partial derivative of an interpolated grid with respect to the ground point, or their sum If $g(X)$ is a polynomial, $\partial g / \partial X$ is straight-forward to compute and, as such, is not detailed in this appendix. If $g(X)$ is an interpolated grid, Appendix A details the computation of $\partial g / \partial X$.

$\partial \Delta I / \partial X^*$ is a 2×3 matrix and will be detailed later in equation (B-6). It will equal zero if the RSM image support data is not adjusted image support data, i.e., if the values of RSM image-space adjustable parameters are all zero. If this is the case, equation (B-2) simplifies to $\partial I / \partial X = \partial g / \partial X$.

$\partial X^* / \partial X$ in equation (B-2) is a 3×3 matrix based on the RSM primary ground coordinate system to Local ground coordinate system transformation. It is evaluated at the unadjusted ground point X, and will be detailed later in equations (B-9) and (B-11).

General form of partial derivatives associated with ground-space adjustable parameters

$$I = h(X, R) = g(X_a)$$

$$\partial I / \partial R = [\partial g / \partial X_a][\partial X_a / \partial R] = [\partial g / \partial X_a][\partial X_a / \partial X_a^*][\partial X_a^* / dR] \text{ or}$$

$$\partial I / \partial R = [\partial g / \partial X_a][\partial X_a / \partial X_a^*][\partial \Delta X^* / dR] \qquad (2 \times m) \qquad \text{(B-3)}$$

Note that $\partial g / \partial X_a$ corresponds to $\partial g / \partial X$ evaluated at $X = X_a$. Similarly, $\partial X_a / \partial X_a^*$ corresponds to $\partial X / \partial X^*$ evaluated at $X = X_a$ (or equivalently, at $X^* = X_a^*$). It is detailed later in equations (B-10) and (B-12). $\partial \Delta X^* / \partial R$ is a $3 \times m$ matrix and will be detailed later in equation (B-7). It is a function of X^* only.

$\partial I / \partial X = [\partial g / \partial X_a][\partial X_a / \partial X^*][\partial X^* / \partial X]$ or

$\partial I / \partial X = [\partial g / \partial X_a][\partial X_a / \partial X_a^*][\partial X_a^* / \partial X^*][\partial X^* / \partial X]$ or

$$\partial I / \partial X = [\partial g / \partial X_a][\partial X_a / \partial X_a^*][Ident + \partial \Delta X^* / \partial X^*][\partial X^* / \partial X] \qquad (2 \times 3), \qquad \text{(B-4)}$$

where "$Ident$" is the 3×3 identity matrix

Note that $\partial \Delta X^* / \partial X^*$ is a 3×3 matrix and will be detailed later in equation (B-8). It will equal zero if the RSM image support data is not adjusted image support data, i.e., if the values of RSM ground-space adjustable parameters are all zero. In which case, $X_a = X$, $X_a^* = X^*$, and (B-4) simplifies to $\partial I / \partial X = [\partial g / \partial X]$.

Detailed supporting partial derivatives

Image-space adjustable parameters

Assume that the RSM adjustable parameters are associated with all twenty possible RSM image-space adjustable parameters ($m = 20$), and that they are labeled $R = [R1 \quad .. \quad R20]^T$. $R1$ through $R20$ correspond to the data indexed by contiguous fields IRO through ICZZ (see the RSMDCA or RSMAPA or RSMECA TRE's). By the definition of each of these adjustable parameters, we have:

$$\Delta I = \begin{bmatrix} R1 + R2x + R3y + R4z + R5x^2 + R6xy + R7xz + R8y^2 + R9yz + R10z^2 \\ R11 + R12x + R13y + R14z + R15x^2 + R16xy + R17xz + R18y^2 + R19yz + R20z^2 \end{bmatrix}$$

where, for ease of notation, x, y, z correspond to the three components of the unadjusted ground point expressed in the Local system, i.e., $X^* = [x \quad y \quad z]^T$. Therefore, differentiating the above, we have:

$\partial \Delta I / \partial R =$

$$\begin{bmatrix} 1 & x & y & z & x^2 & xy & xz & y^2 & yz & z^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & x & y & z & x^2 & xy & xz & y^2 & yz & z^2 \end{bmatrix}, \quad \text{(B-5)}$$

Note that $\partial \Delta I / \partial R$ is a 2×20 matrix since all RSM image-space adjustable parameters are assumed active. If less than all twenty are active, the corresponding matrix columns of the inactive parameters are not present. Typically, three parameters affecting the image row coordinate and three parameters affecting the image column coordinate are active, in which case, $\partial \Delta I / \partial R$ would be a 2×6 matrix.

Now, differentiating ΔI with respect to X^* we obtain:

$$\partial \Delta I / \partial X^* = \quad \text{(B-6)}$$

$$\begin{bmatrix} R2 + 2R5x + R6y + R7z & R3 + R6x + 2R8y + R9z & R4 + R7x + R9y + 2R10z \\ R12 + 2R15x + R16y + R17z & R13 + R16x + 2R18y + R19z & R14 + R17x + R19y + 2R20z \end{bmatrix}$$

The above matrix is a 2×3 matrix. Also, if an image-space adjustable parameter is inactive, simply substitute zero for its value in the above equation.

<u>Ground-space adjustable parameters</u>

Assume now that the RSM adjustable parameters are associated with all sixteen possible RSM ground-space adjustable parameters ($m = 16$), and that they are labeled $R = [R21 \quad .. \quad R36]^T$. $R21$ through $R36$ correspond to the data indexed by contiguous fields GXO through GZZ (see the RSMDCA or RSMAPA or RSMECA TRE's).

By the definition of each of these adjustable parameters, we have:

$$\Delta X^* = \begin{bmatrix} R21 - R25z + R26y + R27x + R28x + R29y + R30z \\ R22 + R24z - R26x + R27y + R31x + R32y + R33z \\ R23 - R24y + R25x + R27z + R34x + R35y + R36z \end{bmatrix}$$

where, for ease of notation, x, y, z correspond to the three components of the unadjusted ground point expressed in the Local system, i.e., $X^* = [x \quad y \quad z]^T$. Therefore, differentiating the above with respect to R, we have:

$$\partial \Delta X^* / \partial R = \begin{bmatrix} 1 & 0 & 0 & 0 & -z & y & x & x & y & z & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & z & 0 & -x & y & 0 & 0 & 0 & x & y & z & 0 & 0 & 0 \\ 0 & 0 & 1 & -y & x & 0 & z & 0 & 0 & 0 & 0 & 0 & 0 & x & y & z \end{bmatrix}, \quad \text{(B-7)}$$

Note that $\partial \Delta X^* / \partial R$ is a 3×16 matrix since all RSM ground-space adjustable parameters are assumed active. If less than all sixteen are active, the corresponding matrix columns of the inactive parameters are not present. Typically, six parameters are active, in which case, $\partial \Delta X^* / \partial R$ would be a 3×6 matrix.

Now, differentiating ΔX^* with respect to X^*, we obtain:

$$\partial \Delta X^* / \partial X^* = \begin{bmatrix} R27 + R28 & R26 + R29 & -R25 + R30 \\ -R26 + R31 & R27 + R32 & R24 + R33 \\ R25 + R34 & -R24 + R35 & R27 + R36 \end{bmatrix} \qquad \text{(B-8)}$$

The above matrix is a 3×3 matrix, and its element values are dependent only on the RSM ground-space adjustable parameter values. Also, if a ground-space adjustable parameter is inactive, simply substitute zero for its value in the above equation.

<u>Coordinate system definitions and transformations</u>

Define the following ground coordinate system representations and transformations:

X_G is the ground position represented in the geodetic RSM primary ground coordinate system;

X_R is the ground position represented in the Rectangular RSM primary ground coordinate system;

X_W is the ground position represented in the WGS-84 rectangular ground coordinate system;

X^* is the ground position represented in the RSM Local ground coordinate system;

$X_W = T(X_G)$ is a nonlinear transformation from geodetic coordinates to WGS-84 rectangular coordinates;

$X_R = AX_W + K1$, where A is a 3×3 matrix and $K1$ a 3×1 vector defined in the RSMIDA TRE as:

$$A = \begin{bmatrix} XUXR & YUXR & ZUXR \\ XUYR & YUYR & ZUYR \\ XUZR & YUZR & ZUZR \end{bmatrix} \text{and } K1 = -A \begin{bmatrix} XUOR \\ YUOR \\ ZUOR \end{bmatrix};$$

$X^* = BX_W + K2$, where B is a 3×3 matrix and $K2$ a 3×1 vector defined (redundantly) in the RSMDCA, RSMAPA, and RSMECA TRE's as:

$$B = \begin{bmatrix} XUXL & YUXL & ZUXL \\ XUYL & YUYL & ZUYL \\ XUZL & YUZL & ZUZL \end{bmatrix} \text{ and } K2 = -B\begin{bmatrix} XUOL \\ YUOL \\ ZUOL \end{bmatrix};$$

$X^* = CX_R + K3$,
where the 3×3 matrix $C = BA^T$ and the 3×1 vector
$K3 = -BA^T K1 + K2$;

$X_R = C^T X^* + K4$,
where the 3×1 vector $K4 = -C^T K3$;

$X^* = BT(X_G) + K2$;

$X_G = T^{-1}(X_W)$, where the superscript "-1" indicates inverse transformation, not the matrix inverse;

$X_W = B^T X^* + K5$,
where the vector $K5 = -B^T K2$;

$X_G = T^{-1}(B^T X^* + K5)$.

Note that using the above definitions, an adjusted ground coordinate can also be defined as follows (where the subscript "a" corresponds to the adjusted ground position):

If the RSM primary ground coordinate system is Rectangular ($X \equiv X_R$) -

$$\begin{aligned} &X_R \to X^* = CX_R + K3 \to X_a^* = X^* + \Delta X^* \\ &\to X_{Ra} = C^T X_a^* + K4 \end{aligned}$$

If the RSM primary ground coordinate system is geodetic ($X \equiv X_G$) -

$$\begin{aligned} &X_G \to X_W = T(X_G) \to X^* = BX_W + K2 \to X_a^* = X^* + \Delta X^* \\ &\to X_{Wa} = B^T X_a^* + K5 \to X_{Ga} = T^{-1}(X_{Wa}) \end{aligned}.$$

<u>Partial derivatives associated with coordinate system transformations</u>

Assuming that the RSM primary ground coordinate system is Rectangular (as specified in the RSMIDA TRE), we have:

$$\partial X^* / \partial X = \partial X^* / \partial X_R = C \qquad (3 \times 3) \qquad \text{(B-9)}$$

$$\partial X / \partial X^* = \partial X_R / \partial X^* = C^T \qquad (3 \times 3) \qquad \text{(B-10)}$$

Note that: $\partial X_a^* / \partial X_a = C$ and $\partial X_a / \partial X_a^* = C^T$ as well, where the subscript "a" corresponds to the adjusted ground position.

Assuming instead that the RSM primary ground coordinate system is geodetic (as specified in the RSMIDA TRE), we have:

$$\partial X^* / \partial X = \partial X^* / \partial X_G = B[\partial X_W / \partial X_G] = B[\partial T / \partial X_G] \qquad (3x3) \qquad \text{(B-11)}$$

$$\partial X / \partial X^* = [\partial X_G / \partial X_W][\partial X_W / X^*] = [\partial T^{-1} / \partial X][(\partial X_W / X^*] = [\partial T^{-1} / \partial X_W]B^T \qquad (3 \times 3) \qquad \text{(B-12)}$$

Note that: $[\partial T^{-1} / \partial X_W]$ is also equal to the matrix inverse $[\partial T / \partial X_G]^{-1}$.

$\partial X_a^* / \partial X_a$ is computed identically as in equation (B-11), except that $\partial T / \partial X_G$ is evaluated at the adjusted ground point value. Similarly, $\partial X_a / \partial X_a^*$ is computed identically as in equation (B-12), except that $\partial T^{-1} / \partial X_W$ is evaluated at the adjusted ground point value.

All that remains is to detail $\partial T / \partial X_G$ and $\partial T^{-1} / \partial X_W$. For ease of notation, let us represent
$X_G = [\lambda \quad \phi \quad h]^T$ and $X_W = [x \quad y \quad z]^T$ in the following.

The transformation T is defined as:

$$\begin{aligned} x &= (N+h)\cos\phi\cos\lambda \\ y &= (N+h)\cos\phi\sin\lambda \quad , \\ z &= ((1-e^2)N+h)\sin\phi \end{aligned}$$

where the prime vertical $N(\phi) = a(1 - e^2 \sin^2 \phi)^{-1/2}$, e is eccentricity, and a is the equatorial radius. All earth constants are WGS-84 constants. Also, it is assumed that $-\pi/2 < \phi < \pi/2$ and $-\pi < \lambda < \pi$.

(The inverse transformation T^{-1} is not needed in order to define the partial derivatives, and as such, is not detailed here. It is a standard transformation, and based on an iterative inverse of the equations defining T.)

Let us make the following abbreviations for ease of notation:

$s\phi \equiv \sin\phi$, $c\phi \equiv \cos\phi$, $s\lambda \equiv \sin\lambda$, $c\lambda \equiv \cos\lambda$;
$P_1 \equiv (N+h)$;
$P_2 \equiv a^{-2}(1-e^2)N^3 + h$.

Differentiating the above equations that define T, using the fact that $\partial N / \partial\phi = ae^2 s\phi c\phi(1-e^2 s^2\phi)^{-3/2} = a^{-2}e^2N^3 s\phi c\phi$, and simplifying we get:

$$\partial T / \partial X_G = \partial X_W / \partial X_G = \begin{bmatrix} -P_1 c\phi s\lambda & -P_2 s\phi c\lambda & c\phi c\lambda \\ P_1 c\phi c\lambda & -P_2 s\phi s\lambda & c\phi s\lambda \\ 0 & P_2 c\phi & s\phi \end{bmatrix} \quad (3x3) \qquad \text{(B-13)}$$

And taking the matrix inverse, we get:

$$\partial T^{-1} / \partial X_W = \partial X_G / \partial X_W = \begin{bmatrix} -(P_1 c\phi)^{-1} s\lambda & (P_1 c\phi)^{-1} c\lambda & 0 \\ -P_2^{-1} s\phi c\lambda & -P_2^{-1} s\phi s\lambda & P_2^{-1} c\phi \\ c\phi c\lambda & c\phi s\lambda & s\phi \end{bmatrix} \quad (3x3) \qquad \text{(B-14)}$$

As a reminder, equations (B-13) and (B-14) are applicable to an unadjusted ground point location (operating point) If the partial derivatives are to be applicable at an adjusted ground point location, utilize the adjusted geodetic ground coordinates, $X_a \equiv X_{Ga} = [\lambda_a \quad \phi_a \quad h_a]^T$, in equations (B-13) and (B-14).

Partial derivatives involving combined RSM image-space and ground-space adjustable parameters

One final comment regarding the RSM adjustable parameters and equations (B-1) through (B-4) - Each of these equations assumed that the RSM adjustable parameters were either all image-space adjustable parameters or all ground-space adjustable parameters. This is the nominal case. However, a mixture can occur, in which case these equations can be combined appropriately in a straight-forward manner. The corresponding RSM adjustable ground-to-image function is presented in equation (B-15). Correspondingly, equations (B-1) and (B-3) can be combined to yield equation (B-16). Similarly, equations (B-2) and (B-4) can be combined to yield equation (B-17).

$$I = h(X,R) = g(X_a) + \Delta I \qquad \text{(B-15)}$$

$$\partial I / \partial R = [\partial g / \partial X_a][\partial X_a / \partial X_a^*][\partial \Delta X^* / dR] + \partial \Delta I / \partial R \quad (2 \times m) \qquad \text{(B-16)}$$

$$\partial I / \partial X = [\partial g / \partial X_a][\partial X_a / \partial X_a^*][Ident + \partial \Delta X^* / \partial X^*][\partial X^* / \partial X] + [\partial \Delta I / \partial X^*][\partial X^* / \partial X] \quad (2x3) \qquad \text{(B-17)}$$